

Online
INTERNATIONAL CONFERENCE/WORKSHOP

ON



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ZERO



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OCTOBER '21 – MARCH '22

ZERO
PROJECT
ONLINE EVENT

An
Interdisciplinary
Perspective

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The Zero Project is a non-profit organization based in the Netherlands. We aim to elucidate the origin of the numeral "zero" in the history of mankind as to date the issue remains unresolved.

Objectives

Compilation of a *Monograph on Zero*, a collection of articles on the origin of zero or else its significance in the field of expertise of the academic concerned.

A proposed associated online *International Conference-cum-Workshop on Zero* in October of 2021, to be co-organized together with Vrije Universiteit, the Netherlands (host) and the Zero Project, where the contributors to the *Monograph on Zero* book project will present and discuss their papers.

Please find the program and list of speakers below, to be augmented by a few more speakers/participants in the run-up to the event.

UNESCO – “After consulting my science colleague I am happy to say that we think that the Zero Project concerns a rich discussion, both mathematical as well as philosophical, with both scientific and heritage components. Very interesting indeed, especially for UNESCO, as it is both the scientific, as well as the cultural organization of the UN. Hence I am delighted to inform you that the Dutch UNESCO Commission is granting Patronage to the organization of the symposium on project Zero. I wish you a lot of success with the Zero project and in particular with the organization of the symposium.”
- Marielies E.W. Schelhaas, Secretary-General, Netherlands Commission for UNESCO

Dr. Robert Lawrence Kuhn - Creator & Host, Closer To Truth (TV), Robert Lawrence Kuhn is a public intellectual; he is the creator, writer, host and executive producer of *Closer To Truth*, the PBS/public television series on cosmos (cosmology/physics), consciousness (brain/mind) and meaning (philosophy of religion) that presents leading scientists, philosophers and creative thinkers discussing fundamental questions (www.closetotruth.com). Dr. Kuhn has written or edited over 30 books, including *The Mystery of Existence: Why is there Anything At All?* (with John Leslie); *Closer To Truth: Challenging Current Belief*; *Closer To Truth: Science, Meaning and the Future*; *How China's Leaders Think* (featuring President Xi Jinping); and *The Man Who Changed China: The Life and Legacy of Jiang Zemin* (China's best-selling book in 2005). Dr. Kuhn is a recipient of the China Reform Friendship Medal, China's highest award. He is chairman of The Kuhn Foundation. He has a BA in Human Biology (Johns Hopkins), MBA (MIT), and PhD in Anatomy/Brain Research (UCLA).

Collaboration with Vrije Universiteit (VU), Amsterdam

The Zero Project looks forward in keen anticipation to the collaboration project on the theme of 'Zero', together with VU Art Curator, Ms Wende Wallert and her team, in VU's new exhibition hall *VU art and science gallery*. The event will coincide with the online *International Conference-cum-Workshop on Zero*, also in collaboration with VU. The concept of zero being axial to a host of cultural disciplines, including the humanities and the sciences, the artistic component showcased in the **0-Exhibition** is an enrichment for VU students, professionals and general public alike.

In addition, we are grateful to VU for substantive engagement with the Zero Project by contributing a paper to the *Monograph on Zero* book project by professor Sharda Nandram, Faculty of Religion and Theology, Beliefs and Practices as well as the prospect of publication, pending peer review, by VU University Press under the guidance of Ms Ilke Jacobs.

Co-organizers (to be determined)

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INTERNATIONAL WORKSHOP ON ZERO / PROGRAM SCHEDULE

Date	Contributor			
	List appended			
October 2021 15:00 CET				
Sun 3	Opening Eric Niehe, Chairperson ZerOrigIndia Foundation UNESCO Robert Kuhn, producer-host <i>Closer to Truth</i> Prof Sharda Nandram, Co-host, VU Wende Wallert, VU Art Curator		0	
Sun 10	Bhaswati Bhattacharya Paul Ernest		1 2	
Sun 17	Brian Rotman (to be confirmed) Jonathan Cender		3 4	
Sun 24	Manil Suri Parthasarathi Mukhopadhyay (to be confirmed)		5 6	
Sun 31	Marina Ville-Petrack Fabio Gironi		7 8	
November 15:00 CET				
Sun 7	Esther Lisa Tischman Erik Hoogcarspel		9 10	
Sun 14	Alexis Lavis Mariana K. Leal Ferreira		11 12	
Sun 21	Anupam Jain Beatrice Lumpkin		13 14	
Sun 28	Amartya Kumar Dutta Joseph Biello		15 16	
December 15:00 CET				
Sun 5	George Gheverghese Joseph Avinash Sathaye		17 18	
Sun 12	John Marmysz Andreas Nieder		19 20	
Sun 19	Djamil Aïssani Esti Eisenmann (to be confirmed)		21 22	
January 2022 15:00				

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Sun 2	Dr T.S. Ravishankar, former Director, Archaeological Survey of India, Mysore Solang UK Vong Sotheara		23 24 25
Sun 9	Sisir Roy (co-author) Venkat Rayudu Posina		26 27
Sun 16	Kaspars Klavins / Marcis Auzins Jeffrey Oaks Jonathan J. Crabtree		28 29 30
Sun 23	Célestin Xiaohan Zhou Daniel Mansfield		31 32
Sun 30	Alberto Pelissero Mayank Vahia, Upasana Neogi		33 34
February 15:00			
Sun 6	Mrs Debra Aczel Dr Miriam Aczel		35 36
Sun 13	Friedhelm Hoffmann Jim Ritter,		37 38
Sun 20	Tiburón Batriedo,		39 40
Sun 27	Ravi Prakash Arya		41
	Sharda Nandram, Ankur Joshi, Puneet Bindlish		42 43 44
	Pending: Alex Jones, Daniel Mansfield, Nathan Sidoli		
	CLOSING		

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=Exhibitions (to be confirmed)

Netherlands, India, Cambodia

AUTHOR BIOGRAPHY / ABSTRACT

-Alphabetical order-

Aczel, Debra - has over 40 years in educational program management, including as program manager at Massachusetts Institute of Technology's Terrascope Program—an interdisciplinary environmental program working to solve pressing global issues. She is co-founder and current co-director of the Amir D. Aczel Foundation for Research and Education in Science and Mathematics, supporting cultural and educational projects in Cambodia. Dr Miriam Aczel is the daughter of Mrs Aczel and Dr Amir Aczel.

Aczel, Miriam - is a Postdoctoral Scholar at California Institute for Energy & Environment (CIEE) at UC Berkeley, working on the Oakland EcoBlock project developing a plan for green energy at the community scale. She is also currently an Honorary Research Associate at Imperial College London's Centre for Environmental Policy. Miriam holds an MSc in Environmental Technology from Imperial College London and a BA in Geography & Earth Systems Science with minors in Italian and Geology & Geophysics from McGill University, Montreal. She has been a visiting researcher at the Environmental Law Institute in Washington, DC, and Tsinghua University in Beijing, China. She currently serves on the editorial board of the Journal of Science Policy and Governance (JSPG) and is Director of Communications for Leaders in Energy, an environmental organization based in Washington, DC. She is co-founder of the non-profit Amir D. Aczel Foundation for STEM Education. Miriam loves exploring new places and learning languages. She speaks French, Italian, and Hebrew and is studying Spanish.

Their presentation discusses the rediscovery of the Khmer Zero in Cambodia by mathematician and author Dr. Amir D. Aczel. Aczel's book, *Finding Zero: A Mathematician's Odyssey to Uncover the Origins of Numbers*. We discuss his search for the origins of numbers that led him on a passionate quest for the oldest representation of zero—a search that took him to India and eventually Cambodia, where he rediscovered an ancient stele with arguably the oldest verified representation of zero yet found. The book—and our presentation—recounts the search and aims to focus attention on the role of Cambodia in the history of mathematics and contribute to redress the long-standing Western bias about the role of Eastern civilizations in mathematical history. This talk also will discuss the connections between mathematics and culture, in this case between the origins of zero and the Indianized culture of Cambodia and how 'telling stories' is an important way to encourage new generations of researchers in mathematics. There are many mysteries around the route that the concept of zero took before finding its way to a temple in the jungles of Cambodia, and there are no Cambodian researchers currently working on the topic, making inclusion of this story in the history of zero significant. It is particularly important to call attention to the topic as Cambodia aims to develop high levels of mathematics education, after the devastation to the education system under the rule of the Khmer Rouge.

Aïssani, Djamil - President, University of Bejaïa, History of Arabic mathematics, Algeria - was born in 1956 in Biarritz (Basque Country, France). He started his career at the University of Constantine in 1978. He received his Ph.D. in 1983 from Kiev State University (Soviet Union). He is at the University of Bejaïa since its opened in 1983/1984. Director of Research, First Head of the Faculty of Science and Engineering Science (1999–2000), Director of the Research Unit *LAMOS (Modeling and Optimization of Systems)*, <http://www.lamos.org>, Scientific Head of the Doctoral Computer School (2004 - 2011), President of the Learned Society GEHIMAB (History of Sciences in the Maghreb and Mediterranean area), he has taught in many universities. Prof. Aïssani was the president of the National Mathematical Committee (Algerian Ministry of Higher Education and Scientific Research, 1995–2005).

Numeration in the Scientific Manuscripts of the Maghreb

In this paper, we begin by recalling the beginning of the mathematics of the Islam's Countries, particularly by emphasizing the influence of Indian arithmetic. In a second step, we present the particularity of the mathematics of the Muslim West (Maghreb and al-Andalus), by revealing the specificity of the digits and the used symbolism. Thirdly, we focus on the role played by the city of Bejaïa (Algeria) in the "popularization" of *Arabic numerals* in Europe, following the stay of the Pisan's mathematician Leonardo Fibonacci. The contribution of this article concerns the presentation of the numeration available in the Maghreb on the basis of the analysis of "*Afniq n'Ccix Lmuhub*" (*Khizana* - scholarly library of manuscripts of *Sheikh Lmuhub*). Discovered at *Tala Uzrar* in 1994, it is currently the only library of manuscripts cataloged in Kabylia (Algeria).

Arya, Ravi Prakash - Prof. Ravi Prakash Arya is a Chair Professor on Maharshi Dayanand Saraswati Chair (UGC) in Maharshi Dayanand University, India. He has to his credit 75 research papers and 70 books running into 84 volumes on the various aspects of Vedas, Vedic Sciences, Vedic Exegesis, Vedic Philology, Vedic Philosophy, Religion, Indian History and Culture. He is the Chief Editor of a quarterly Research Journal 'Vedic Science' dealing with the scientific interpretation of Vedas & Allied literature and ancient Indian traditions. He is also the editor of the annual 'World Vedic Calendar'. He has convened several national and international conferences/seminar/workshops on Vedic Sciences, Ayurveda, Yoga, Spirituality and Indian knowledge system.

Zero in Ancient Sanskrit Literature of India

The invention of zero has proved a game changer in the human history of science and technology. No scientific advancement would have been possible without the invention of zero. In fact, its invention revolutionized everything in science and technology. The famous mathematician G.B. Halsted and noted French mathematician and astronomer during the eighteenth century give credit of the invention of zero to Indians. Although, the presence of zero has been traced to many ancient civilizations including India, but the question of its origin is still a mute point among historians and mathematicians. As such, in the present monograph, a humble attempt will be made to gather data from the various ancient Indian Vedic and Post Vedic Sanskrit texts to understand the origin of zero; the philosophy behind its origin; various denominations of zero used in various phases of Indian civilization; and the philosophy and epistemology behind the symbolic representation of zero as (0).

Bhattacharya, Bhaswati - Fellow, University of Göttingen, Germany, is a historian affiliated with the Centre for Modern Indian studies, Göttingen University as a senior research fellow. Her recent publications include a monograph on the Indian Coffee House *Much Ado over Coffee: Indian Coffee House Then and Now* (SocialScience Press 2017). She is the co-editor of 'Politics of Advertisement and Consumer Identity: The Making of the Indian Consumer' (Routledge, in Press). Dr Bhattacharya is working on another monograph on the marketing and consumption of coffee in India.

Viewing the Zero as a Part of Cross-cultural Intellectual Heritage

Currently, there are different claimants for the honour of inventing the zero as a digit, a part of the decimal system. These claims are made on the basis of the evidence found in one particular country. None of the claims has, however, been able to convincingly rule out the rival claims. This paper suggests that research along the terms of cross-cultural history or connected history linking the threads of information coming from different civilizations regarding the knowledge of the zero may yield a different, border-crossing biography of the zero.

Biello, Joseph – Applied Mathematics, University of California at Davis, USA, is a Professor of Mathematics at the University of California, Davis. He began his career studying astrophysical fluid dynamics, but gradually moved to fluid dynamics of a more mundane nature - the Earth's atmosphere. His mathematical expertise lies at the interface of partial differential equations and asymptotic methods, and his perspective on the concept of zero is informed by ideas of dominant balance that arise from asymptotics. He lives with his wife and two young daughters in California. While on sabbatical in 2012, Biello and his family, by stroke of serendipity, found themselves renting an apartment above Dr. Rene Samson. From this have sprung many collaborative explorations into fluid dynamics and mathematics, including a contribution to the present Monograph.

The Significance to Physics of the Number Zero

Although the numeral zero is ubiquitous in physics, there are certain "environments" in physics where zero takes pride of place. This is illustrated by focusing on Conservation Laws, Symmetry and non-linear dynamics. In all of these subjects, the zero fulfils the role of an anchor point, the eye at the centre of the hurricane, so to speak. We try to make these connections clear in a language that is accessible to non-experts. We hope to make clear that although many mathematical models are so complex that they defy even the most powerful computers, exact or approximate results deriving from Conservation or Equilibrium reveal many characteristics of these complex systems. [Note: coauthor Dr René Samson, and chairperson of the Zero Project, passed away in July of 2019. The *Monograph on Zero* is dedicated in his honor.]

Bindlish, Puneet – coauthor, see Nandram, Sharda - is Assistant Professor, Hindu Spirituality at the Faculty of Religion and Theology, Vrije Universiteit Amsterdam, The Netherlands. Earlier he taught at the Department of Humanistic Studies, Indian Institute of Technology (BHU) Varanasi, India. He brings a blend of academic, consulting and entrepreneurial experience

across healthcare, telecom, technology, banking & insurance, education, sports, public sectors. His fields of interest are: Spirituality, Leadership, organisational behavior, entrepreneurship and integrative intelligence.

Cender, Jonathan – Independent researcher, unaffiliated

A New Zero Arising from Sunya

Zero changed. The number zero used today changed as it passed through adoptive cultures outside its origins in India. Today's zero based on the empty set does not represent the generative sunya exemplified by Nagarjuna's Philosophy of Void. After briefly examining salient changes, a new number zero debuts based not on the empty set but on an "absent" set defined in terms of some specified set of numbers. The absent set becomes generative through the operation of division. This leads to a major development in number systems built through arithmetic operations (i.e., the Real and Complex numbers) since the generation of "array numbers" as unique quotients make possible an infinite number of dimensions. Space can be filled through arithmetic operations which is not the case now. Various issues addressed include the uniqueness of zero, a reciprocal for zero that is not also a multiplicative inverse of zero, and maintenance of the arithmetic of zero familiar to us.

Crabtree, Jonathan J. - Historian of Mathematics, Melbourne, Australia - Having initially studied economics at the University of Melbourne, he is an autodidact, studying the history of mathematics since 1983. His first paper was on the history of Euclid's definition of multiplication whilst his next paper explored the writings of Descartes and Newton. Another paper presented in Hungary spanned the writings of Diophantus, Cardano, Euclid, Liu Hui and Brahmagupta. Having reviewed original writings in Latin, Greek, Arabic and Sanskrit, Crabtree has also written magazine articles on mathematics history and been a guest lecturer on the history of zero at Indian universities.

Notes on the Absence of Brahmagupta's Definition of Śūnya and the Need to Rebuild Elementary Mathematics from Zero. The entry for 'zero, n. and adj.' in the online Oxford English Dictionary (updated June 2018), contains approximately 9300 words. However, there are zero references to either the Sanskrit word śūnya or Brahmagupta's 7th Century mathematical definition of śūnya (zero). Elsewhere we find definitions of zero falsely attributed to Brahmagupta such as the result of subtracting any number from itself (Barrow 2001: 38). This 'nothing remaining as a result of subtraction' and placeholder notion may have been an idea that reached the Arabic world on its way to Europe. Yet, as will be discussed, Brahmagupta defined zero via addition, not subtraction and the physical foundations of mathematics are a zero-sum game.

Dhital, Vishwanath - is Assistant Professor, at Department of Humanistic Studies, Indian Institute of Technology (Banaras Hindu University), Varanasi, India. Post Doctorate: ICPR fellow, SVDV, BHU, Varanasi and Doctorate, Masters, Bachelors: Sampurnanand Sanskrit University, Varanasi. Early Sanskrit education: Mahesh Sanskrit Gurukulam, Devghat, Nepal. He has a traditional background in Indian Philosophy especially in Nyaya-Vaisheshika Philosophy. He is honoured by the President of India for his contribution to Sanskrit Language. He has published about 10 Books and Papers.

Dutta, Amartya Kumar - Theoretical Statistics and Mathematics Division, Indian Statistical Institute, Kolkata, India, is a Professor of Mathematics at the Indian Statistical Institute (ISI), Kolkata. He obtained his PhD in Mathematics under Prof. S.M. Bhatwadekar at the Tata Institute of Fundamental Research, Mumbai, in 1994 and has several research publications in Commutative Algebra and Affine Algebraic Geometry, especially Affine Fibrations. Since 2000, Prof. Dutta has been regularly delivering lectures and writing articles on topics in History of Science, especially on Ancient Indian Mathematics. He is the recipient of the first Satish C Bhatnagar Award in History of Mathematics of the Indian Mathematical Society. Prof. Dutta has been teaching history of mathematics in ancient India and science in modern India in the one-year Indology course at the Ramakrishna Mission Institute of Culture, Kolkata.

Zero in the Decimal System: a few stray thoughts

Much has been written on the importance of the decimal notation and one of its crucial components: the zero. In this article, alongside a brief revisit of the mathematical ideas in the decimal notation and its known history, we discuss the mathematical sophistication of the Sanskrit number-vocabulary, the likely precursor to the decimal notation, which gets overlooked in discourses on the decimal system and zero. One of the unique contributions of ancient Indian mathematicians is their conceptualization of zero as a number and their introduction of binary operations with zero. We shall discuss this algebraic use of zero in the general context of the emergence of Algebra as a distinct discipline in ancient Indian mathematics. We shall also briefly discuss the cultural nuances of the Sanskrit terms for zero and highlight a few instances of the use of the mathematical zero as a metaphor in Sanskrit literature and philosophy.

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Ernest, Paul - Emeritus Professor, Exeter University, UK

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Nought Matters: the mathematical and philosophical significance of zero

This is an exploration of the mathematical and philosophical significance of zero. It looks at the mathematics of zero, and the history of its development from three perspectives: syntactic (signs), semantic (meanings), and pragmatic (social use and contexts). It draws a clear distinction between nothing and zero, which avoids various paradoxes and contradictions. There is an analysis of the meaning of mathematical symbols, especially zero. After sketching various histories it entertains the hypothesis that a concept of the empty void is a necessary prerequisite for the development of proper zero (as a number not just a placeholder) in the history of numeration. This is confirmed positively in Indian and Mayan cultures and negatively in Ancient Egypt. Above all, it celebrates zero as essential in mathematics and digital culture. It credits Brahmagupta, at around CE 628, as the Indian analogue of Euclid with regard to integer arithmetic (\mathbb{Z}), but repudiates the Great Human Theory of history which attributes advances to key individuals. It fails to resist making the odd joke about being, such as nought matters.

Gironi, Fabio - Unaffiliated, was Humboldt Research Fellow at the University of Potsdam, Germany. He works on the history of American philosophy, epistemology, and contemporary realism. He has published numerous articles on Wilfrid Sellars, realism in contemporary continental philosophy, and the philosophy of science. He is the author of *Naturalizing Badiou: Mathematical Ontology and Structural Realism* (Palgrave, 2014) and editor of *The Legacy of Kant in Sellars and Meillassoux: Analytic and Continental Kantianism* (Routledge, 2017).

Zero and Śūnyatā: likely bedfellows

The aim of this paper is to highlight some conceptual connections between the concept of zero and the philosophical concept of śūnyatā. Both ideas first arose in the Indian subcontinent, and I will try to suggest that this geographical congruence is more than a coincidence. As such, this will likely be a more speculative contribution to this volume than others, but I will attempt to substantiate my proposal by stressing the conceptual advantages deriving from such a syncretic enterprise. I will begin with an overview of the historical development of the number and concept of zero, followed by some considerations of its treatment in contemporary mathematics, as well as an examination of its less evident semiotic properties. I will then proceed to re-examine the concept of śūnyatā, identifying its central ontological significance in the philosophy of Nāgārjuna's. In this context, I will examine the idea of the 'trace' in the thought of Jacques Derrida, viewing it as another actor in the assemblage of concepts which I attempt in this paper. Finally, I undertake a synthetic reading of these three 'voids'—of zero, of śūnyatā, and of the trace—aimed at a speculative rearrangement of these heterogeneous concepts, in a philosophical exercise which I see as both a possible and necessary evolution of 'comparative philosophy'.

Hertog, Irma den - (www.irmadenhertog.nl), is a visual artist. Graduated from the Royal Academy of Art in The Hague. She also studied art history at Leiden University and graduated in 1990 on *Regressive Progression. An Inquiry into Inspiration as Mystical Death. As a return to the Origin. Cosmic Void. Zero and Infinity at the same time*. In 1997 she was confronted with this experience herself. Since then, this mystical Death as Death by Fire has been central to her work as a visual artist. In addition, as a scientist she conducts research into cross-linkages between art, mysticism and philosophy. Working title of that manuscript is: *Eros, Sacred Fire and Divine Madness*.

Hoffmann, Friedhelm - Professor Friedhelm Hoffmann, Ludwig-Maximilian University of Munich (LMU), Germany

Zero in ancient Egypt

On these pages, I would like to give a very short summary of 3000 years of Egyptian conceptions of non-existence/emptiness/zero mainly in mathematical contexts. This is not at all new, rather, I can make use of my 2004/5 paper in *Enchoria*.

Hoogcarspel, Erik - philosopher, unaffiliated, the Netherlands, studied mathematics, philosophy, phenomenology, buddhology, indology and religious anthropology at the Universities of Groningen and Leiden. He lectured in different institutions among others the University of Nijmegen and the Internationale School Voor Wijsbegeerte. Apart from articles, he published 'The Central Philosophy, Basic Verses', an annotated translation from Sanskrit into English of the second century text called the *Mūlamadhyamakakārikāḥ*, written by the Indian Buddhist philosopher Nāgārjuna (2005); a Dutch translation, called 'Grondregels van de filosofie van het midden' (2005); 'Phenomenal emptiness' (2016), suggesting to use

phenomenology as a method for better understanding and developing Buddhism in the West. Presently he is doing research into the origins and peculiarities of the concept of self in the history of Western philosophy.

From Emptiness to Nonsense: the Constitution of the Number Zero

The subject will be introduced by a review of a paper written by Professor Andreas Nieder: “Representing Something Out of Nothing: The Dawning of Zero”. This will be followed by a reflection on the precise nature of the transformation of zero from a signifier of quantity to a number. Edmund Husserl calls this the constitution (Stiftung) of the zero as a new ideal object and he described the process in his “The Origin of Geometry”. Zero as a number is an element of a self-contained symbolic deductive system, that does not refer to the life world and has no sense at all, it has become non-sense. This implies that no meaning can possibly be transferred from zero as a quantity to zero as a number. The history of zero as a number coincides with the history of abstract mathematics and the development of calculi, the tremendous usefulness of which is precisely based on its very meaninglessness. All speculation about any eventual progress of the concept of zero as a quantity to zero as a number is therefore unsubstantiated. Marc Richir has shown that the constitution of an ideal object is a jump, it leaves a gap between its new sense and its historical basis. The paper will conclude with the question whether there is a link between this historical basis of zero as a quantity and the concept of emptiness in Buddhism, this supposed relation has been and still is the pet of quite a few Buddhologists.

Jain, Anupam, is presently Professor of Mathematics and Principal (I/C) Govt. College, Sanwer (Indore) India. Previously he served as Professor and Head in the Department of Mathematics and Controller- Exams in Govt. Holkar (Autonomous) Science College, Indore. He got his Ph.D. from Meerut University (Presently C.C.S. University) in 1992 on the topic ‘Contribution of Jainacharyas to the Development of Mathematics’. He has written 18 Books, over 100 research Papers on History of Mathematics and over 40 on various aspects of Jainism together with 163 general articles. He has been honored by various awards since 1993. Some of them are Shastri Parishad Journalism Award (1993).

Significance of Zero in Jaina Mathematics

The term Jaina Mathematics represents the mathematical concepts and ideas available in Jaina Canonical and Non-canonical literature. The literature related to Karaṅṅnyoga section of Digambara Jaina tradition and Gaṅṅṅnyoga section of Āvetāmbara Jaina tradition have a lot of content of Mathematician’s interest. Division of numbers in the three groups Saṅṅhyāta (Countable), Asaṅṅhyāta (Un-countable) and Ananta (Infinite) is available in ancient Jaina literature. The definition of Utkāṅṅta saṅṅhyāta (maximum countable) can be given only with the help of zero. Without the knowledge of numbers and zero it is not possible to define Utkāṅṅta saṅṅhyāta and other related quantities. We have not seen any inscription prior to 500 C.E. and having use of zero in Jaina tradition. But we find the use of zero in mathematical texts Bakhshālī Manuscript, texts of Digambara Jaina tradition – a²akhaṅṅdāgama, Pancāstikāya Mahābandha, Lokavibhāga, Tiloyapaṅṅṅ and Dhavalā Commentary etc. and text of Āvetāmbara Jaina tradition like Sthānāṅṅa sūtra, Anuyogadvāra sūtra, Āvaṅṅyaka Nirukti, Bāhātṅṅetra Samāsa etc. in mathematical sense at many places. All such references are explored and analysed in the paper to give a comprehensive picture before the academic world. It indicates that the use of zero and the decimal place-value system was popular in Jaina tradition during 1st-2nd century C.E.

Joseph, George Gheverghese - School of Mathematics, University of Manchester, UK, was born in Kerala, Southern India, and lived in India until he was nine. His family then moved to Mombasa in Kenya where he received his schooling. He studied at the University of Leicester and then at Manchester. His teaching and research have ranged over a broad spectrum of subjects in applied mathematics and statistics. In recent years, however, his research has been mainly on the cultural and historical aspects of mathematics with particular emphasis on the non-European dimensions to the subject and its relevance for mathematics education.

The Enormity of Zero

A few years ago on a British television programme I was asked: “Why did Zero originate in India?” Fortunately, I was allowed enough time to develop an answer without assuming as most television programmes do today that the audience watching have the attention span of a grasshopper. Trying to gather my thoughts, I resorted to the familiar ploy of taking refuge in definitions. If zero merely signified a magnitude or a direction separator (i.e. separating those above the zero level from those below the zero level), the Egyptian zero, nfr, dating back at least four thousand years, amply served these purposes. If zero was merely a place-holder symbol, indicating the absence of a magnitude at a specified place position (such as, for example, the zero in 101 indicates the absence of any “tens” in one hundred and one), then such a zero was

already present in the Babylonian number system long before the first recorded occurrence of the Indian zero. If zero was represented by just an empty space within a well-defined positional number system, such a zero was present in Chinese mathematics a few centuries before the Indian zero. The absence of a symbol for zero did not prevent it from being properly integrated into an efficient computational tool that could even handle solution of higher degree order equations involving fractions. However, the Indian zero alluded to in the question was a multi-faceted mathematical object: a symbol, a number, a magnitude, a direction separator and a place-holder, all in one operating within a fully established positional number system. Such a zero occurred only twice in history –the Indian zero which is now the universal zero and the Mayan zero which occurred in solitary isolation in Central America around the beginning of Common Era.

Joshi, Ankur – coauthor, see Nandram, Sharda - is working as Assistant Professor at FMS-WISDOM, Banasthali Vidyapith and Visiting Faculty at Vrije University, The Netherlands. He is a Fellow of MDI-Gurugram in Public Policy and Governance, where he did doctoral research on Gurukul education system. At Banasthali he contributes to research, training and teaching through OMRISE Research Group (collaboration with Buurtzorg and Praan Group), National Center for Corporate Governance (supported by NFCG, New Delhi) and National Resource Center (setup by Ministry of Education, Government of India). He has developed online modules, conducted various training sessions for teachers, civil servants and students. He has published about 20 papers and book chapters.

Klavin, Kaspars - International Institute of Indic Studies, University of Latvia, Prof. Dr. hist., is a specialist in Middle Eastern history; Islamic civilization; East Asian and Korean history, culture and spirituality; medieval history and Baltic history. Recently he has published a book on East-West intercultural relations (*The “Other” and the “Self”: Supplement to East-West Cross-Cultural Studies*. Riga: University of Latvia Press, 2020). Klavins has been a visiting professor at Roosevelt University (USA) through the Fulbright Program, an adjunct research associate at Monash University (Australia), a research associate in the Department of Archaeology at the University of Reading (UK), a visiting professor at the University of Münster (Germany), a professor of history and cross-cultural management at the Emirates College of Technology in Abu Dhabi (United Arab Emirates), a professor in the Faculty of Engineering Economics and Management at Riga Technical University (Latvia) and a visiting professor at Pusan National University (South Korea). Klavins is currently an acting professor in the Department of Asian Studies at the University of Latvia (UL), a member of the Scientific Council of the UL International Institute of Indic Studies, the project director at the UL Centre for Korean Studies and the president of the Baltic Association of Korean Studies (BAKS).

Challenges in Interpreting the Invention of Zero

Attempts to explain the importance of inventing zero and the application thereof in the context of the history of culture, philosophy and the exact sciences bring to the forefront a range of problems in relation to the overlapping of challenging issues (and terminology) in science and culture. The symbol of zero is qualified both as a scientific discovery (or invention) and an empirically found solution for the satisfaction of certain practical needs of humans. Moreover, its invention is also based on references to certain religious and philosophical teachings. All this makes the explanation of this phenomenon extremely difficult, taking into account that explanations frequently connect fundamentally different fields that actually address completely different areas of research. Furthermore, when looking more closely into the materials related to scientific-technical and religious-philosophical explanations, we see that the emergence of zero (or the idea of “emptiness”, “nothingness”) on the religious-philosophical side does not at all indicate the existence of this phenomenon in exact-scientific use, and vice versa. There might be a discussion about “emptiness” or “nothingness” within a specific historical period in a certain society, and at the same time zero might not show up at all in the mathematical theory and practice known to that society, as was the case, for example, in medieval Europe. Likewise, the introducers of zero in mathematical practice may theoretically disagree with the idea of “nothingness” per se at the religious-philosophical level, as was seen in the Europe of the Early Modern Period.

Lavis, Alexis – Ph.D. Philosophy and Sanskrit Studies (University of Normandy & EPHE), Professeur Agrégé de Philosophie, former assistant/lecturer in ancient philosophy and comparative studies at University of Normandy, former lecturer at Sciences Pô Paris in religious studies. Presently, Associate Professor of Philosophy (History of Western Philosophy & Comparative Philosophy) at Renmin University of China (Beijing).

The influence of Buddhism on the Invention and Development of Zero

Since recent datings of Bakhshālī Manuscript confirm our understanding that the zero is indeed an Indian creation, what is it in Indian culture which could have influenced an invention of this kind? From data relating to the Bakhshālī Manuscript, it is possible to recognise the influence of Buddhism, and from this, to go on to try to see how this movement could have played a role in the theory leading to the inception of the placeholder zero. Moreover, with Brahmagupta this first form and use of zero was developed to the point of becoming a number, and thus adopting an operative role. While the writer of the Brāhmasphuṭasiddhānta was not a Buddhist, it is still possible to show in that document how Buddhist thinking, particularly its speculative developments in the meaning of śūnyatā as described by the great writers of Mahāyāna Buddhism, may provide a favourable setting for theories about the numerical concept of zero, even to the point where it is perceived as a condition of its “thinkability” or conceptualisation

Lumpkin, Beatrice – Assoc. Prof. Malcolm X College, Chicago City Colleges, USA, became a mathematics teacher after years of work as an electronics technician, technical writer and labor organizer. She earned an M. S. in Mathematics Education at Northeastern University and an M. S. in Mathematics at Illinois Institute of Technology. As a mathematics professor at Malcolm X College, Chicago City College, also with high school teaching experience, Lumpkin has consulted with schools and authored books on the multicultural history of mathematics.

The Number Zero in Ancient Egypt

Throughout history, cultures around the world have independently developed distinct number systems and distinct systems of recording numerals. However, racist, Eurocentric theories of history devalued the achievements of black and brown peoples. Mathematical work outside of Europe is little known or acknowledged. In pre-Columbian Peru and Central America, astronomers and mathematicians developed positional-value systems of numerals that used different methods to indicate an empty or zero-valued position. Others, as in Ancient Egypt, used an additive numeral system without positional, also called place value. Without place value, there is no need for a zero as a placeholder. Ancient Egyptian mathematics did use zero as a number in surviving bookkeeping records. Also labels on vertical guidelines, still visible at pyramid and mastaba construction sites, show the use of a system of directed numbers with a zero-reference level. The same symbol, used to show a zero remainder in bookkeeping, is used to label the zero level at Ancient Egyptian construction sites. The concept of infinity was also explored in the papyri. The concept of zero as a number, as well as a concept of infinity, could have developed in the course of the extensive Egyptian exploration of series of numbers.

Mansfield, Daniel - School of Mathematics and Statistics, University of New South Wales, Australia

Marmysz, John – Philosopher, College of Marin, California, USA, holds a Ph.D. in philosophy from SUNY Buffalo and teaches at the College of Marin in Kentfield, California. His research focuses on the issue of nihilism and its cultural manifestations. Marmysz is the author of *Laughing at Nothing: Humor as a Response to Nihilism* (SUNY Press, 2003), *The Path of Philosophy: Truth, Wonder and Distress* (Wadsworth, 2011), *The Nihilist: A Philosophical Novel* (No Frills Buffalo, 2015), *Cinematic Nihilism: Encounters, Confrontations, Overcomings* (Edinburgh University Press, 2017) and is coeditor (with Scott Lukas) of *Fear, Cultural Anxiety and Transformation: Horror, Science Fiction and Fantasy Films Remade* (Lexington Books, 2009).

The Fear of Nothingness in the West

The fear of nothingness has deep roots in the West. Whereas Eastern “emptiness” is commonly associated with spiritual peace and creative potential, in the West, nothingness is more commonly associated with complete nonexistence, oblivion and the extinction of all value and meaning. In this regard, Westerners have traditionally conceived of nothingness as a dreadful and terrifying lack; something to be overcome and defeated rather than something to be embraced. The roots of the Western fear of nothingness can be traced at least as far back as the Presocratics and their philosophical efforts to conceptualize an eternal, immutable, uncreated and stable substance out of which all things emerge. Despite the varied and ephemeral nature of the world’s appearances, the Presocratics suggested that there remains something stable, permanent and dependable underneath it all. Whether it be Thales’ claim that “all is water,” Anaximander’s claim that the universe arises from “Apeiron,” or Democritus’ assertion that everything comes from atoms, the strategy pursued by these ancient Greek thinkers served to offer the comfortable assurance that our cosmos has a steady and knowable foundation. The universe ultimately rests on one “thing” rather than on nothing at all. In setting this precedent, the Presocratics influenced later Western philosophers, whose concerns concentrated on establishing fixed and substantial foundations for the world, while also repudiating systems of thought emphasizing the primacy of nothingness. Such systems came to be criticized as “nihilistic”; a moniker intended to highlight negativity and meaninglessness. It is only in recent times that Western thinkers have started to reassess this appraisal, coming to find something life-affirming in nihilism and in the experience of nothingness itself. This paper examines nihilism and the fear of nothingness in Western philosophy, from its origins in Presocratic philosophy, to its reassessment in contemporary Western thought.

Mukhopadhyay, Parthasarathi - Associate Professor of Mathematics at Ramakrishna Mission Residential College, India, he is a Gold Medalist M.Sc. in Pure Mathematics, has done his M.Phil. in Topological Algebraic Structures and Ph.D. in Algebraic Theory of Semirings, from the University of Calcutta. He has published several research papers and visited a number of Universities and Institutes in India and abroad delivering over 150 invited lectures on his areas of interest that includes History of Mathematics. He has jointly authored several text books on school Mathematics, on Abstract Algebra, edited a book on Linear Algebra and translated *The Man Who Knew Infinity* in Bengali.

From Śūnya to Zero — an Enigmatic Odyssey

Who invented 'zero'? Well, interestingly, there is no one-liner answer. And the acceptability of any attempted answer generally depends on the perception of the seeker, as the connotation of the word 'zero' can be perceived from several different but interrelated perspectives. Many ancient civilizations, including India, had their own version of 'zero' or zero-like concept or symbol as a representative of 'nothingness'; some as a philosophical conundrum, elsewhere some others even in a practical sense, such as a filler or a gap on their counting board; but except in India, none of these early and somewhat hesitant initial concepts did ultimately mature to its true mathematical potential. Today it is generally accepted worldwide that this peerless concept of a decimal place-value system of enumeration in tandem with the true 'zero' of our present-day mathematics, evolved in ancient India. Embracing all these paradigms of claims and counterclaims about the origin of 'zero' made till date, this article is a nutshell version of an odyssey, an unparalleled journey from a concept to a number in its own right, perhaps the most significant creation of human mind ever.

Nadram, Sharda - coauthors: Ankur Joshi, Puneet Bindlish –

Full Professor Hindu Spirituality and Society at the Vrije Universiteit Amsterdam and an Associate Professor at the Nyenrode Business University in the field of Business and Spirituality. She has earned two Bachelors and two Masters at the University of Amsterdam: one in Psychology and the other in Economics. She has graduated from her PhD in Social Sciences at the Vrije University in Amsterdam. She is an adjunct professor at Banasthali University in Jaipur, India. Her fields of interest are: Hinduism and Spirituality, Business and Spirituality, Entrepreneurial behavior, organisational innovation and integrative intelligence.

Zero, One and Infinity: An Integrative spiritual perspective

The world of numbers keeps us intrigued for one reason or the other. From negative numbers to imaginary numbers and from various constants to peculiar ratios, they never cease to surprise us. And our curiosity and imagination keeps on inspiring us to look for these patterns in nature. Consequently leading the way for discoveries that are important to not just meeting our material goals but also spiritual ones at the same time. However, due to an extraordinary reliance of modern applied science on numbers, we have generated more scholarly interest, especially in the last few centuries, around the application of numbers to the material realm as compared to the spiritual. Amidst our attention to the peculiar or special numbers and ratios, there are some numbers or rather concepts that still look in our face, inviting us to explore a whole new world of possibilities all over again. Though these concepts laid the foundation of modern day mathematics as we use it, yet at times we take them for granted and are not usually interested in revisiting our understanding about their nature, their past and future possibilities. In this chapter, we revisit three of these foundational concepts - Zero, One and Infinity (ZOI) from an integrative spiritual perspective.

Neogi, Upasana – coauthor, see **Vahia, Mayank** - is pursuing the Advanced Diploma in VLSI from the Centre for Development of Advanced Computing. She has M. Sc. in Electronic Science from Dinabandhu Andrews College, affiliated to University of Calcutta; worked with Megatherm Electronics Pvt. Ltd., Kolkata as a Research and Development Engineer. She pursued a post-graduation Diploma in Astronomy and Planetarium Science; did a six months project on Radioastronomy at the Giant Metrewave Radio Telescope, Khodad. She also worked in Archeo-astronomy and did a project at Tata Institute of Fundamental Research; and also worked as a Research Assistant at Jawaharlal Nehru University, Delhi.

Nieder, Andreas - Director, Institute of Neurobiology, University of Tübingen, Germany - studied Biology at the Technical University Munich, Germany. In 1999, he received his Ph.D. degree from the Rheinisch-Westfälische Technical University in Aachen, Germany. He then moved to the Massachusetts Institute of Technology, Cambridge, USA, to carry out postdoctoral research on the neural basis of numerosity judgments in monkeys. From 2003 to 2008, he worked as a junior research group leader at the Hertie-Institute for Clinical Brain Research & Dept. of Cognitive Neurology of the University of

Tübingen, Germany. Since 2008 he is Professor of Animal Physiology at the Department of Biology at the University of Tübingen, Germany, where he is also the director of the Institute of Neurobiology. He is interested in how higher brain centers of humans, monkeys and corvids enable intelligent, goal-directed behaviors in general, and numerical competence in particular.

A Sense for “Nothing”

Zero is a magic number. It represents emptiness, nothing - and yet it is considered one of the greatest cultural achievements of mankind. For a brain that has evolved to process sensory stimuli (“something”), conceiving of empty sets (“nothing”) as a meaningful category demands high-level abstraction. Recent studies from cognitive neuroscience now provide an insight into how an abstract concept like zero can emerge. Both in behavioral and neural processing, the emergence of zero passes through four stages. In the first stage, the absence of a stimulus, “nothing”, corresponds to a (mental/neural) resting state lacking specific signature. In the second stage, stimulus absence is grasped as a meaningful behavioral category, but is still devoid of quantitative relevance. In the third stage, “nothing” acquires a quantitative meaning and is represented as empty set at the low end of a numerical continuum or number line. Finally, the empty set representation is extended to become the number zero, thus becoming part of a combinatorial number of symbols system used for calculation and mathematics. The concept of zero shows how our minds and brains, originally evolved to represent stimuli, detach from empirical properties to achieve ultimate abstract thinking. Because of this, the story of zero tells us a great deal about the mind leaving empirical grounds and raising to new intellectual heights.

Oaks, Jeffrey - University of Indianapolis, Indiana, USA

Pelissero, Alberto - Department of Humanities, University of Turin, Italy, is Full Professor of Indology at the Department of Humanities at Torino University (Italy), where he is referent for the Indological curriculum of the PhD program in Humanities and teaches Philosophies and Religions of India and Central Asia and Sanskrit Language and Literature. Most recent publications in English: *Metaphysical Landscape, Interior Landscape: Two Variants of Mythical-Religious Geography within Ancient Indian Worldview* ("Journal of Oriental Research" 2017), *The Epistemological Model of Vedantic Doxography according to the Sarvadarśanasamgraha for the Study of Indian Philosophy* ("Annali di Cà Foscari" 2020).

Some More Unsystematic Notes on śūnya

The paper is an overview of the themes regarding the concept of *śūnya* “void” in Indian thought, mainly *mādhyaṃika*, with reference to *apoha* semantic theory, and in relation with another couple of problematic relationships, the metaphoric use of words (diffused in the milieu of poetics) and the paradox of ineffability (current in vedantic brahmanical circles). Did the mathematical, the grammatical, or the Buddhist philosophical meaning of *śūnya* come first? The paper tries to illustrate the great deal of debate current on this question, but does not offer a unique answer.

Posina, Venkata Rayudu – coauthor, see Roy, Sisir - has a long-standing commitment to the development of the science of consciousness, which propelled him to pursue wide-ranging experimental and theoretical investigations of the brain, mind, and cognition at Harvard Medical School (Boston), National Brain Research Centre (Delhi), Salk Institute for Biological Studies (La Jolla), and National Institute of Advanced Studies (Bengaluru). His research findings were published in *Nature, Mind & Matter, Neuron, Neuroscience, Proceedings of the National Academy of Sciences USA*, and *Progress in Brain Research*, among other scientific journals. Currently, in joint work with Professor Sisir Roy, he is characterizing the geometry and algebra of consciousness. Using the category theory of mathematical modelling of mathematical objects, he is building a mathematical representation of consciousness.

Ravishankar, T.S. - former Director, Archaeological Survey of India, Mysore, had his M.A. (Sanskrit), M.A. (Ancient His. & Arch) and Ph.D. from the University of Mysore. He joined the Epigraphy Branch of the Archaeological Survey of India where he eventually served as Head of the Branch, meanwhile retired as the Director (Epigraphy). He is the Chairman of Epigraphical Society of India Vice-President of South Indian Numismatic Society. Dr. T.S. Ravishankar established his expertise in deciphering Sanskrit Inscriptions. He coordinated and conducted many workshops on Paleography, Epigraphy and Numismatics at Delhi and Mysore. He has also participated in many important archaeological excavations and contributed a number of scholarly articles in English and Kannada languages to many reputed journals.

Reflections on early dated inscriptions from South India

The study of the transition from the numerical system of writing to the decimal system of writing, including the quest to date the origin of zero as a place numeral, has actively engaged the attention of modern researchers and scholars alike. The subject is in fact so vast, gripping, as well as baffling that it will most likely continue keep them engaged for quite a while to come. There is no need to reiterate the fact that India is very rich in epigraphical wealth. Hundreds and thousands of inscriptions have been reported from across the length and breadth of the country, written in various languages and in varied scripts, and following different dating systems. It is because of this abundance and the sheer complexity of these inscriptions, coming from different parts of India and belonging to different dynasties, that it has become an arduous task to pinpoint when exactly the numerical system of writing gave way to the decimal system in India, and when zero first made its appearance in Indian inscriptions. Further, the chronological sequencing of the changes in the system of writing – from numerical to decimal - that had taken place over time is also difficult to establish. In this paper, while keeping the vastness of the subject in mind, an attempt has been made to reexamine some of the early dated inscriptions that have come to light in the last few years, both as part of the reports from archaeological excavations and from epigraphical explorations. Efforts have also been made to connect these new findings with the hypotheses of and the views expressed by earlier scholars. The goal of this exercise was to ascertain and confirm the presence and usage of decimal systems of writing, and also the use of zero as a placeholder, in South India. Thus, the primary task undertaken has been to review some of the already known dated inscriptions from Andhra Pradesh, Karnataka, Kerala and Tamil Nadu.

Ritter, Jim – is a retired professor of mathematics and history of science at the University of Paris 8. His research interests lie in the study of rational practices in Ancient Egypt and Mesopotamia and in the history of twentieth-century general relativity and unified theories.

Standing in place: The zeroes of Mesopotamia

The metrological origins of (sexagesimal) positional notation in Ancient Mesopotamian number writing and the material and bureaucratic constraints in its scribal practice required from the beginning a way to deal with the problem of ‘zero’, in the sense of an empty place-holder. We shall examine the various solutions found—there were at least four—during the three millennia that literate Mesopotamian civilization existed.

Roy, Sisir - National Institute of Advanced Studies, IISc Campus, Bangalore, is a theoretical physicist, Visiting Professor and Senior Homi Bhabha Fellow, National Institute of Advanced Studies, IISc Campus, Bangalore. Previously he was Professor, Physics and Applied Mathematics Unit, Indian Statistical Institute, Kolkata during 1993-2014. He worked as Distinguished Visiting Professor in many US and European Universities. His main field of interests includes Foundations of Quantum Theory, Theoretical Astrophysics, Brain Function Modeling and higher order cognitive activities. He published more than one hundred seventy papers in various peer-reviewed international journals, twelve monographs and edited volumes. His recent books include: Decision making and modeling in cognitive science (Springer).

Category Theory and the Ontology of Sunyata

Notions such as Sunyata, Catuskoti, and Indra’s Net, which figure prominently in Buddhist philosophy, are difficult to readily accommodate within our ordinary thinking about everyday objects. Famous Buddhist scholar Nagarjuna considered two levels of reality: one called conventional reality and the other ultimate reality. Within this framework, Sunyata refers to the claim that at the ultimate level objects are devoid of essence or “intrinsic properties”, but are interdependent by virtue of their relations to other objects. Catuskoti refers to the claim that four truth values, along with contradiction, are admissible in reasoning. Indra’s Net refers to the claim that every part of a whole is reflective of the whole. Here we present category theoretic constructions which are reminiscent of these Buddhist concepts. The universal mapping property definition of mathematical objects, wherein objects of a universe of discourse are defined not in terms of their content, but in terms of their relations to all objects of the universe is reminiscent of Sunyata. The objective logic of perception, with perception modeled as [a category of] two sequential processes (sensation followed by interpretation), and with its truth value object of four truth values, is reminiscent of the Buddhist logic of Catuskoti. The category of categories, wherein every category has a subcategory of sets with zero structure within which every category can be modeled, is reminiscent of Indra’s Net. Our thorough elaboration of the parallels between Buddhist philosophy and category theory can facilitate better understanding of Buddhist philosophy, and bring out the broader philosophical import of category theory beyond mathematics.

Sathaye, Avinash - Department of Mathematics, University of Kentucky, USA, was born in India where he met his would-be professor Shreeram Abhyankar. Under his inspiration, Avinash proceed to do a Ph. D. in Algebra and Algebraic

Geometry at Purdue University. He had studied Sanskrit as a child and developed an interest in Classical Indian philosophy as well as Mathematics. He has been working at the University of Kentucky, Lexington KY USA for last 45 years.

Division by Zero (Khahara) in Indian Mathematics

Since Brahmagupta (seventh century), the reciprocal $1/0$ of the number 0 was used in mathematical operations. The ideas were not restricted to the notions of limits in Calculus, but appear to be unusual algebraic constructs, which have no parallel in history of mathematics. We present an analysis using ideas from modern algebra. We also discuss numerous similar structures in modern mathematics.

Sidoli, Nathan Camillo - Waseda University, School for International Liberal Studies, Japan

Sotheara, Vong - Royal University of Phnom Penh, History Faculty (Epigraphy), Cambodia

Suri, Manil - Department of Mathematics and Statistics, UMBC, USA, is a distinguished university professor of mathematics at the University of Maryland, Baltimore County. His field of research is finite element methods. He has been involved in several outreach projects to raise the cultural profile of mathematics, including co-writing the play, *The Mathematics of Being Human*, which was staged at different venues in 2014-2016. He is also the author of the novels, *The Death of Vishnu*, *The Age of Shiva* and *The City of Devi*, which form a trilogy exploring the present, past and imagined future of India. His fiction has won several awards and has been translated into 27 languages. As a contributing opinion writer for *The New York Times*, he has written several op-eds on mathematics, India and gay rights. He is in the process of completing a new book, *How to Build the Universe Using Only Numbers*, which explains mathematics to non-mathematicians.

Putting a Price on Zero

In the fall semester of 2017, I was teaching a course on the history of mathematics to a class consisting entirely of mathematics majors. News of the Bakhshālī manuscript carbon dating came just as we were discussing the origins of zero. To motivate the class, I had them go through an exercise: suppose royalties had been imposed on every use of zero (like for a patentable discovery), how should the money be divided? It was a cute exercise, and gave rise to some interesting larger questions, so based on the feedback, I decided to write an opinion piece for the *New York Times*.

The piece was published soon after our class discussion. Right afterwards, I started receiving a barrage of comments by Twitter and email in response. This was a touchier subject than I'd imagined – it seemed to have struck a nerve for some readers. It was also a great teaching moment, since I was able to discuss the comments with the class. In the next section, I present the article, and in the final section, very briefly summarize some reader comments and my class's responses to them.

Tishman, Esther Lisa – Ph.D., BCC, Associate Professor Emerita, University of Oregon

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The Zero Triumphant: Allegory, Emptiness and the Early History of the Tarot

In the early modern period, what we now call Tarot was known as the game of "trionfi" (i.e. triumphs). The game was not imbued with esoteric import; it was just a card game, but a game embedded in the history of allegory. The Tarot emerges in 15th-century Italy as what appears to be an amalgamation of (a) four-suited playing cards brought into Europe via the Mamluk empire from the Muslim near East -- with (b) a series of allegorical images originating in medieval Christian allegory. In this amalgam of straightforward number cards (the Mamluk suits) and allegorically imagined trump cards, we find a fascinating new game that collates the arithmetic logic of counting, with the Western sublimating logic of allegory. It is here that the Zero enters the scene: one of the earliest tarot decks (the 1491 Sola-Busca) labels the Fool card with a Hindu zero digit. This use of the Hindu numbering system emerges despite the fact that the other trump cards employ Roman numerals. The Sola-Busca deck thus very straightforwardly demonstrates the ways in which East meets West within the history of Tarot. More precisely, the Sola-Busca deck suggests the impact of the zero digit, and the Indian conception of *sunyata* or emptiness that it conveys, upon Western allegorical thought. In a brief coda discussing the figure of the Fool in 16th-century European literature, this essay ultimately suggests that the Tarot deck helps transform a medieval allegorical world view into a distinctly early modern interpretive mode.

Uk, Solang - translator of *Find Zero*, by the late-professor Amir Aczel - obtained a Bachelor's and a Master's degree at Louisiana State University in Baton Rouge in 1966 and 1968, respectively, and his Ph. D. in insecticide toxicology. In 1972 he joined the Ciba-Geigy Agricultural Aviation Research Unit (AARU) based at Cranfield Institute of Technology (CIT), England. He has lectured on the aerial control of the migratory locust in Nairobi, Kenya (April 1975) and in Karachi, Pakistan (April 1976), organized by the FAO Locust Control and Emergency Operations, Plant Protection Division, Rome,

and attended by crop protection specialists of the ministry of agriculture of Algeria, Egypt, Ethiopia, Jordan, India, Iran, Mali, Sudan, Tunisia, and Turkey. He retired in 1998, having published 50 scientific articles and four books.

Revisiting Khmer Stele K-127

A stone stele code-named K-127 was discovered in 1891 on the east bank of the Mekong River in Sambaur district of Kratié province, Cambodia by the French governor of the province. A recent search to relocate the ancient temple where the stele was found showed only brushes and a piece of plain rock of similar colour to K-127. French epigraphist, George Cœdès translated its inscriptions in 1930 and was astonished to find that it bears the number 604 of the śaka era which is equivalent to 682 CE. This is the first Khmer inscription that shows the date in numeral form. The top right corner of the stele was broken when it was found, and the first eight lines of the inscriptions were partly broken off on the right side. The zero on K-127 is two centuries older than the internationally accepted oldest Indian zero found in the city of Gwalior. In 1931 Cœdès published an article to debunk the belief that our modern decimal number system with the numeral zero as place holder originated in Europe from the Greeks. The origin of the Indian zero comes from the Buddhist logic of *catuskoti* or tetralemma. To underline the logic, a brief discussion between the wandering ascetic Vacchagotta and Buddha is presented. During the Cambodian civil war in the 1970s, the short lived Cambodian republican government moved many ancient artefacts to warehouses of Angkor Conservation in Siem Reap for safe keeping. When the Khmer Rouge came to power in April 1975, nobody knows what happened to K-127. It was presumed lost. In 2013, an American mathematics professor who had been travelling the world in search of the origin of the numeral zero in our decimal number system, learned about K-127. He set off again on his search through Southeast Asia, and eventually found K-127 in a shed of Angkor Conservation. He organized with the ministry of Culture and Fine Arts to have the stele brought back to the National Museum in Phnom Penh. In July 2018, the Bodleian Library at Oxford University in England announced in newspapers and through YouTube the results of their C_{14} analysis of the Bakhshali manuscript found in 1881 in Bakhshali village near Peshawar in today's Pakistan. The age of the manuscript from the C_{14} test spans from the third to the tenth century CE. The results were immediately rejected by five experts in Indian mathematics, historians of Indian math, and a specialist in Indian palaeography. So long as the age of the Bakhshali manuscript remains unsettled, K-127 can rightfully claim to bear the world oldest zero.

Vahia, Mayank N., coauthor Neogi, Upsana (see above) - is the Dean of a newly instituted School of Mathematical Sciences, Narsee Monjee Institute of Management Studies, Mumbai. He recently retired as professor at the Tata Institute of Fundamental Research, Mumbai which he joined in 1979. He spent the first 25 years building space-based telescopes for US, Russian and Indian satellites. In recent years, he has become interested in understanding the origin and growth of astronomy and science in India. He is also interested in issues related to science and society. He has led Indian efforts in the Astronomy Olympiad with consistently good results. He has published 250 research papers and written or edited 5 books. His biggest love is talking to students of all ages. He is a fellow of the National Academy of Sciences, India; International Astronomical Union, International Union of Pure and Applied physics, Royal Astronomical Society, Institute of Physics (UK) etc. He credits his achievements to TIFR and to his wife Neeta and daughters Mimansa and Durva.

ZERO: In Various Forms - Mayank Vahia and Upasana Neogi

Zero makes its appearance in several apparently unrelated ideas. The Babylonians and the Mayans discovered zero independently. The Babylonians used zero as a placeholder or separator, whereas the Mayans used zero in their calendrical systems. The advancement of zero from the concept of placeholder to a more abstract entity representing the absence of anything first occurred in India. Since then the idea of zero has had several manifestations. Here we summarize some of the ideas of zero as a null entity.

Vahia, Mayank N.

Connecting Zero's

As various articles in this volume demonstrate, zero has many forms and finds expression in a whole host of context, each as mystifying as the other. Each culture seems to have a very specific reaction to the number and the reaction varies significantly from culture to culture. From the great hatred in Europe to the near obsession of their cousins in the Indian subcontinent the entire spectrum of reactions can be found. Egyptians, Sumerians and the Chinese managed to do some interesting algebra while circumventing zero completely. In this chapter we will investigate why cultures even with deeply common roots reacted so differently to zero. We take the example of the Indo-European cultures and their diametrically opposite reactions to zero. We suggest that this may be related to their experience of nature post separation which were further amplified by their way of life. We then study the wider context of zero in its different manifestation and explore the

relation between the different forms of zero. We show that Music seems to make a connecting link between these different aspects of zero.

Ville, Marina - University of Tours, France - Modern mathematics

Can we divide by zero?

Not dividing by zero is one of the strongest prohibitions that we learn as children. Yet, as we grow in age and in competence, we realize that many mathematical constructions would be much simpler if we could divide by zero. But this implies enlarging the family of numbers and making space for an infinite number, ∞ , as in the Riemann sphere, or even many infinite numbers, as the founders of calculus envisioned.

Zhou, Célestin Xiaohan - Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing, China, Assistant Research Fellow, got his Ph.D. from Laboratoire SPHERE (Sciences, Philosophy, History, UMR 7219), University Paris Diderot, France in 2018. At present, he mainly works on the continuities between mathematical writings from the 13th century to the 16th century in ancient China and he is also interested in the historiography of sciences since the 19th century in terms of mathematics in ancient China.

On the placeholder in numeration and the numeral zero in China

In China, a decimal system of numeration is reflected in the numbers of oracle bones script of the 14th century to the 11th century BCE. The place value system is clearly shown in the numbers on coinage of the 8th century to the 3rd century BCE. The decimal place value system was completely established in the counting rods numeration, a description of the use of which is recorded in Mathematical Canon of Sun Zi of the late 3rd century and the beginning of the 4th century, and the earliest written counting rods numbers exist in a Dunhuang manuscript around the 10th century. The procedures of multiplication, division or extraction of roots show that precise vacant positions in the number are emphasized and strictly respected by the ancient practitioners. In the 13th-century mathematical works, written symbols \bigcirc is systematically used as a placeholder in number. The “procedure of the positive and the negative” is usually regarded as general rules of addition and subtraction between positive, negative numbers and zero. But this procedure should be interpreted in the context of mathematical method fangcheng. General rules concerning the process of elimination numbers in which zero is involved are clearly expressed and adopted, even though there is no explicit declaration of the existence of numeral zero. zero is not explicitly regarded as a number.

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